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**CERTIFIED PUBLIC ACCOUNTANT**  
**FOUNDATION LEVEL 1 EXAMINATION**  
**F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE**  
**METHODS**  
**DATE: THURSDAY 30, MAY 2024**  
**MARKING GUIDE AND MODEL ANSWERS**

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## Marking guide

### QUESTION ONE

#### Kigali Ltd (KL)

*Marks*

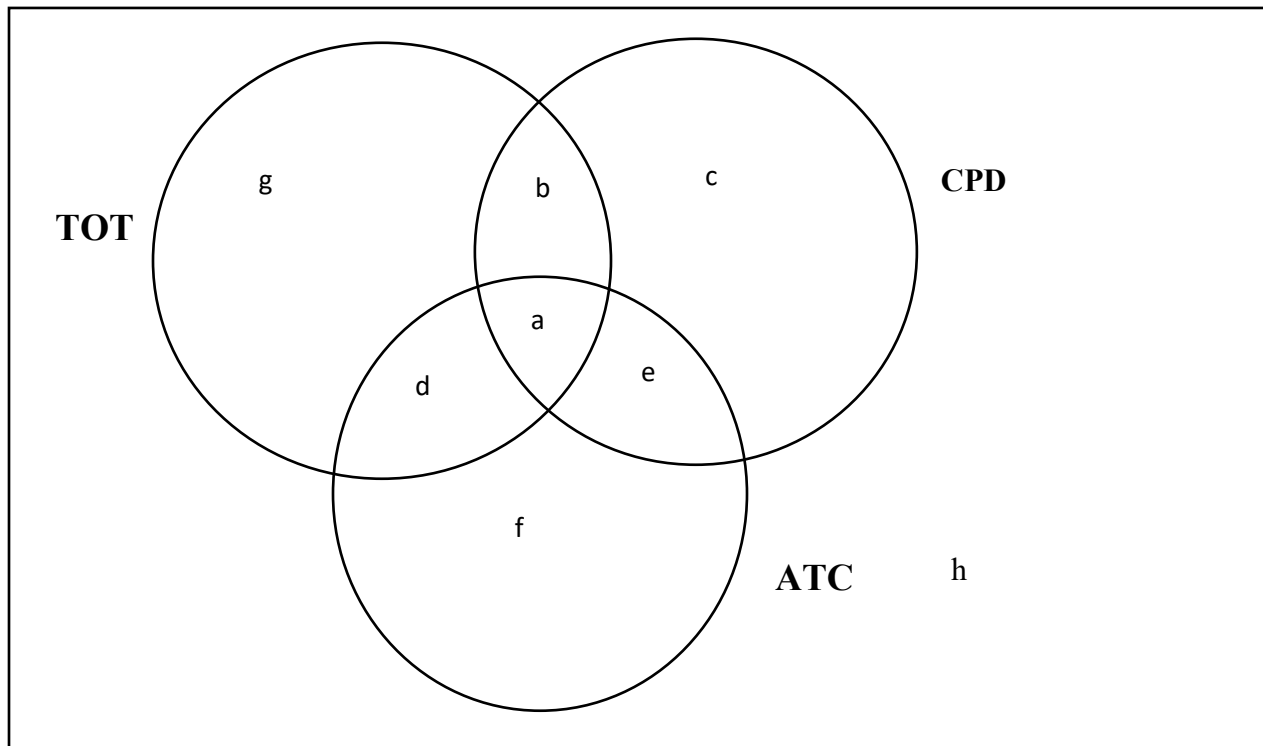
a) Difference between overlapping sets and equal sets 1 Mark each	2
b) (i) Correct allocation of letters from a to h in the Venn diagram	8
(ii) 1 Mark for correct addition and 1Mark for correct answer	2
(iii) 1 Mark for correct addition and 1Mark for correct answer	2
(iv) 1 Mark for correct addition and 1Mark for correct answer	2
(v) Applications (1 Mark each, max 4)	4
<b>Total marks</b>	<b>20</b>

#### Model Answer

a) Overlapping sets are those which have some elements in common while equal sets refer to the sets that have the same elements.

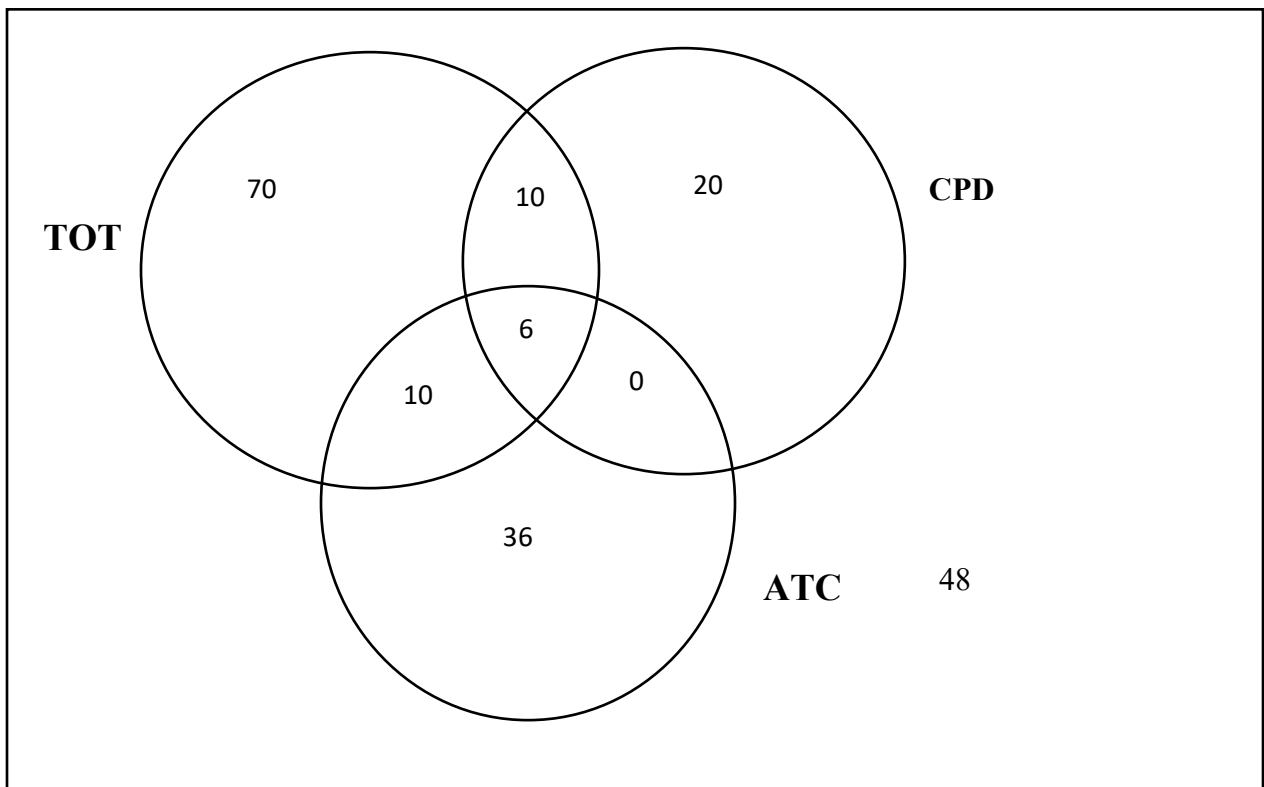
#### b) Representing the information on a Venn diagram

Workings:



▪  $a+b+c+d+e+f+g+h=200$

- $a+b+d+g=96$        $a+b+c+e=36$      $a+d+e+f=52$
- Here  $a=6$              $a+d=16$        $d=16-6$      $d=10$
- $a+b=16$                $b=16-6$        $b=10$
- $a+b+d+g=96$        $6+10+10+g=96$      $g=70$
- $a+e=6$                  $e=0$
- $a+d+e+f=52$          $f=52-6-10-0$      $f=36$
- $a+b+e+c=36$          $6+10+0+c=36$      $c=20$
- $a+b+c+d+e+f+g+h=200$      $6+10+20+10+0+36+70+h=200$      $h=48$



ii) The number of members who attend TOT but did not attend CPD is 80 (70+10) this is the intersection of TOT and ATC plus those who attended TOT only.

iii). The number of members who attend ATC and CPD but did not attend TOT is zero (0), this is the intersection of ATC and CPD

iv) The number of members who attended none of the above is the value of “h” which is 48 members

v) Applications of set theory

- It is used in capturing statistical data.
- It is used in solving counting problems
- It shows the logical relationship between two or more sets.
- It creates a basis for probability theory
- It is a research tool that can be used in data capturing

**Marking guide**

**QUESTION TWO**

*Marks*

a) (i) Calculation of lower quartile	
Calculation of class boundaries (0.5 Marks each, max 3.5)	3.5
Calculation of cumulative frequency (0.5 Marks each, max 3.5)	3.5
Formula for lower quartile	0.5
Computation of lower quartile	0.5
(ii) Characteristics of mean deviation (1 Mark each, max 3)	3.0
<b>Maximum marks</b>	<b>11</b>
b) i) Computation of correlation coefficient	
Calculation of the totals for each column in the table (0.5 marks each, max 3)	3.0
Stating the formula for correlation coefficient	1.0
Calculation of the correlation coefficient	2.0
Interpretation of the result	1.0
(ii) Computation of coefficient of determination	1.0
Interpretation of the result	1.0
<b>Maximum marks</b>	<b>9.0</b>
<b>Total marks</b>	<b>20</b>

## Model Answer

a) i) Lower quartile of the data

Class	Class boundaries	Frequency	Cumulative frequency
20-24	19.5-24.5	9	9
25-29	24.5-29.5	17	26
30-34	29.5-34.5	21	47
35-39	34.5-39.5	18	65
40-44	39.5-44.5	15	80
45-49	44.5-49.5	9	89
50-54	49.5-54.5	11	100

$$\text{Lower quartile} = L_1 + \left( \frac{\frac{n}{4} - cfb}{f_l} \right) * Cw$$

**L<sub>1</sub>**: lower class boundary of the lower quartile class

**F<sub>1</sub>**: Frequency of lower quartile class

**Cfb**: Cumulative frequency of a class before lower quartile class

**N**: number of observations

**Cw**: class width

$$\begin{aligned} \text{Lower quartile} &= 24.5 + \left( \frac{\frac{100}{4} - 9}{17} \right) * 5 \\ &= \mathbf{29.2} \end{aligned}$$

ii) **Mean deviation** is the measure of dispersion that gives the average absolute difference between each item and the mean. It should be noted that the mean deviation is a much more representative measure than the range since all item values are considered in its calculation.

### Characteristics of mean deviation

- ✓ It is a good representative measure of dispersion that is easy to understand. It is, therefore, useful for comparing the variability between distributions of like nature.
- ✓ The modulus sign makes it impossible to handle the mean deviation Theoretically and this limits its applicability for advanced analysis.
- ✓ When the mean is not a whole number, its computation is rather complicated.

- ✓ Because of the modulus sign, the mean deviation is virtually impossible to handle theoretically and thus is not used in more advanced analysis

b) GATARE cell

i) **Correlation coefficient.**

a) **Computation of correlation coefficient**

SN	Residents	Age (x) in years	Weight (y) in kilograms	x <sup>2</sup>	y <sup>2</sup>	xy
1	A	9	14	81	196	126
2	B	5	10	25	100	50
3	C	40	65	1600	4225	2600
4	D	30	43	900	1849	1290
5	E	25	38	625	1444	950
6	F	15	20	225	400	300
7	G	17	40	289	1600	680
8	H	12	15	144	225	180
9	I	22	45	484	2025	990
10	J	38	56	1444	3136	2128
<b>Total</b>	<b>10</b>	<b>213</b>	<b>346</b>	<b>5817</b>	<b>15200</b>	<b>9294</b>

i) 
$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2} \times \sqrt{n \sum y^2}}$$

$$r = \frac{(10 \times 9,294) - (213 \times 346)}{\sqrt{(10 \times 5817) - (213 \times 213)} \times \sqrt{(10 \times 15,200) - (346 \times 346)}}$$

$$r = \frac{92,940 - 73,698}{113.14 \times 179.68}$$

$$r = \frac{19,242}{20,329} = 0.95$$

**Interpretation:** The correlation coefficient is 0.95 which means that there is a strong correlation (relationship) between age and weight for the residents in GATARE cell.

ii) Solve for the coefficient of determination for the data provided from the survey and interpret your results.

Coefficient of determination  $R^2 = (r)^2$

$$R^2 = (0.95)^2 = 0.90 \text{ which is approximately } 90\%$$

Interpretation: 90% variation in weight is explained by the linear relationship between weight and age. Or 90% variation in weight can be explained by the variation in the ages of the residents and 10% cannot be explained by the variation in age but by other factors.

### Marking guide

#### QUESTION THREE

Marks

a) Uses of index numbers (1 marks each, max 3)	<b>3.0</b>
b) i) Computation for commodity and interpretation (1 marks each, max 4)	<b>4.0</b>
ii) Calculation of Laspeyre's price index	
Calculation of $P_0Q_0$ for each commodity (0.5 marks each, max 2)	2.0
Calculation of $P_1Q_0$ for each commodity (0.5 marks each, max 2)	2.0
Calculation of the totals for $P_0Q_0$ and $P_1Q_0$ (0.5 marks each, max 1)	1.0
Stating the formula for computing Laspeyre's price index	0.5
Computation of Laspeyre's price index	0.5
<b>Maximum marks</b>	<b>6.0</b>
c) (i) Calculation probability less than FRW 600,000	1.0
(ii) Computation of $P$ ( $FRW\ 600,000 \leq x \leq FRW\ 800,000$ )	2.0
(iii) Computation of $P$ ( $FRW\ 800,000 \leq x \leq FRW\ 1,000,000$ )	2.0
(iv). Calculation probability more than FRW 1,200,000	2.0
<b>Maximum marks</b>	<b>7.0</b>
<b>Total marks</b>	<b>20</b>

### Model Answer

a) Uses of index numbers

- The price index numbers are used to measure changes in a particular group of prices and help us in comparing the movement in prices of one commodity with another.
- Index numbers of industrial production provide a measure of change in the level of industrial production in a country.
- The quantity index numbers show the rise or fall in the volume of production, volume of exports and imports etc.

- The import and export price indices are used to measure the changes in the terms of trade of a country.
- Index numbers are also used to forecast business conditions of a country and to discover fluctuations and business cycles.
- Index numbers are also used to forecast business conditions of a country and to discover fluctuations and business cycles.

b)

i)

### Price index numbers for the given commodities

Price index number = price of the commodity for the current year

$$\text{Price index number} = \frac{\text{price of the commodity for the current year}}{\text{price of the commodity for the previous year}} \times 100$$

- Banana; Price index =  $\frac{700}{500} \times 100 = 140$

Interpretation: There was a 40% increase in prices of bananas from 2019 to 2021.

- Potatoes; Price index =  $\frac{350}{450} \times 100 = 78$

Interpretation: There was a 22% decrease in prices of potatoes from 2019 to 2021.

- Mangoes; Price index =  $\frac{1,000}{900} \times 100 = 111$

Interpretation: There was an 11% increase in prices of mangoes from 2022 to 2023.

- Oranges; Price index =  $\frac{800}{600} \times 100 = 133$

Interpretation: There was a 33% increase in prices of oranges from 2022 to 2023.

ii) Laspeyre's price index

Year		Commodity				Total
		Bananas	Potatoes	Mangoes	Oranges	
2022	P0	500	450	900	600	
	Q0	50	90	150	100	
	P1	700	350	1,000	800	



<b>2023</b>	<b>Q1</b>	70	110	120	80	
	<b>P0Q0</b>	25,000	40,500	135,000	60,000	<b>260,500</b>
	<b>P1Q0</b>	35,000	31,500	150,000	80,000	<b>296,500</b>

Laspeyre's price index

$$\text{Laspeyre's price index} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$\text{Laspeyre's price index} = \frac{296,500}{260,500} \times 100 = 113.82$$

c) Normal distribution

(i) Less than FRW 600,000

$z = \frac{x - \mu}{\sigma}$  where z is a z score or z value for probability,  $\sigma$  is standard deviation,  $\mu$  is the mean income and x is the requirement.

Here  $x = \text{FRW } 600,000$ ,  $\mu = \text{FRW } 800,000$  and  $\sigma = \text{FRW } 200,000$

$P(x < \text{FRW } 600,000) = ?$  Change x into z

$$P\left(z < \frac{\text{FRW } 600,000 - \text{FRW } 800,000}{\text{FRW } 200,000}\right) = P\left(z < \frac{-\text{FRW } 200,000}{\text{FRW } 200,000}\right)$$

$$P(z < -1) = 0.1587 = 15.87 \approx 16\%$$

(This value is obtained from the normal distribution table)

The probability of people with the income less than FRW 600,000 is 16%

(ii) Between FRW 600,000 and FRW 800,000

$P(\text{FRW } 600,000 \leq x \leq \text{FRW } 800,000) = ?$  Change x into z

$$P\left(\frac{\text{FRW } 600,000 - \text{FRW } 800,000}{\text{FRW } 200,000} \leq z \leq \frac{\text{FRW } 800,000 - \text{FRW } 800,000}{\text{FRW } 200,000}\right)$$

$$P(-1 \leq z \leq 0) = P(z = 0) - P(z = -1)$$

$$= 0.5000 - 0.1587 \text{ (From normal distribution table)}$$

$$P(-1 \leq z \leq 0) = 0.3413 = 34.13\% \approx 34\%$$

The probability of people with the income between FRW 600,000 and FRW 800,000 is 34%

(iii) Between FRW 800,000 and FRW 1,200,000

$P(\text{FRW } 800,000 \leq x \leq \text{FRW } 1,000,000) = ?$  Change  $x$  into  $z$

$$P\left(\frac{\text{FRW } 800,000 - \text{FRW } 800,000}{\text{FRW } 200,000} \leq z \leq \frac{\text{FRW } 1,200,000 - \text{FRW } 800,000}{\text{FRW } 200,000}\right)$$

$$P(0 \leq z \leq 2) = P(z = 2) - P(z = 0)$$

$$= 0.9772 - 0.5000 \text{ (From normal distribution table)}$$

$$P(-1 \leq z \leq 0) = 0.4772 = 47.72\% \approx 48\%$$

The probability of people with the income between FRW 800,000 and FRW 1,200,000 is 48%

(iv) More than FRW 1,200,000

$P(x \geq \text{FRW } 1,200,000) = ?$  Change  $x$  into  $z$

$$P\left(z \geq \frac{\text{FRW } 1,200,000 - \text{FRW } 800,000}{\text{FRW } 200,000}\right)$$

$$P(z \geq 2) = 1 - P(z = 2)$$

$$= 1 - 0.9772 \text{ (From normal distribution table)}$$

$$P(z \geq 2) = 0.0228 = 2.28\% \approx 2.3\%$$

The probability of people with the income more than FRW 1,200,000 is 2.3%

## Marking guide

### QUESTION FOUR

*Marks*

d) Properties of binomial experiment (1 marks each, max 4)	<b>4.0</b>
e) (i) Stating the formula	2.0
Computation for probability	2.0
(ii) Stating the formula of expected number	2.0
Computation for expected number	2.0
Stating the formula of standard deviation	2.0
Computation for standard deviation	2.0
(ii) Stating the formula	2.0
Computation for probability	2.0
<b>Maximum marks</b>	<b>16</b>
<b>Total marks</b>	<b>20</b>

## Model Answers

### d) Properties of a binomial experiment

A binomial experiment or a Bernoulli trial is a probability experiment that has the following properties:

- There must be number of repeated identical trials,  $n$ .
- Each trial results into two possible outcomes referred to as either success or failure.
- The trials are independent i.e. the outcome on one trial does not affect the outcome on the other trial.
- The probability of success is the same on every trial.

**Note:** The probability of success is denoted by  $p$  while the probability of failure is denoted by  $q$  Where  $q=1-P$ .

b) i) Our tax periods are binomially distributed, let  $P$  denotes probability

$$P(\text{not audited}) = 0.20$$

$$P(\text{audited}) = 0.80$$

$$N = 200$$

$$P(x) = {}^n C_x P^x Q^{n-x}$$

$$P(x=7) = {}^{200} C_7 (0.2)^7 (0.8)^{200-7}$$

$$P(x=7) = 3176716400 * 0.0000128 * 0.0000000842498 = 0.250$$

ii) The mean (Expected number)  $= n * p$

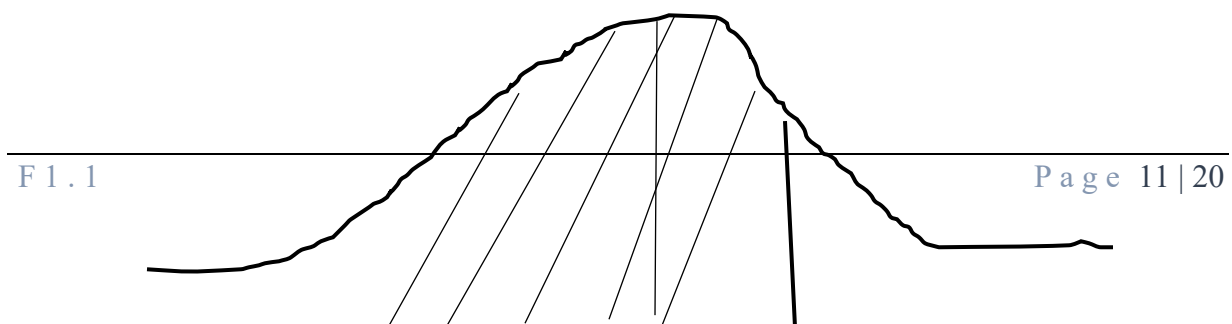
$$\text{Mean} = 80 * 0.2 \quad \text{Mean} = 16$$

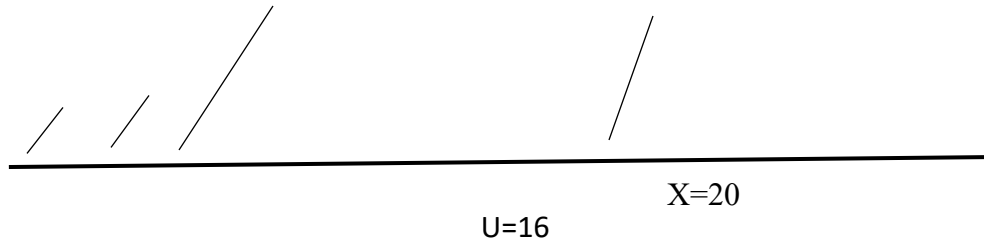
$$\text{Standard deviation} = \sqrt{npq}$$

$$= \sqrt{80 * 0.2 * 0.8}$$

$$= 12.8^{1/2} \quad \text{Standard deviation} = 3.5777$$

iii) The probability that at most 20 of the tax periods will not be audited is calculated as follows:





At most 20 tax periods will end up not audited

$$Z = \frac{X-u}{\text{standard deviation}} \quad z = (20-16)/3.5777 \quad z=1.11$$

$$P(X \leq 20) = 0.5 + 0.3665 \\ = 0.8665$$

### Marking guide

#### QUESTION FIVE

*Marks*

a) Definitions	
Correct definition from i) to iv) (1 Mark each, max 4)	<b>4.0</b>
b) Game theory	
i) Table showing row minimum and column maximum	2.0
Table resulting into application of dominance rule	2.0
Computation of the percentage time played by Union (U1, U2)	2.0
Identification of the best strategy played by Union	1.0
Computation of the percentage time played by UR (S1, S2)	2.0
Identification of the best strategy played by Union	1.0
ii) Value of the game and its interpretation	
Computation of the value of the game	1.0
Interpretation of the result	1.0
<b>Maximum marks</b>	<b>12</b>
c) Merits of purpose sampling	
Merits (1 mark each, max 4)	<b>4.0</b>
<b>Total marks</b>	<b>20</b>

### Model Answers

**a) i) Saddle point** refers to the equilibrium situation where the agent tries to minimize the maximum possible loss or maximizing the minimum gain. It happens to games that have pure strategy.

A saddle point in a payoff matrix is the one which is the smallest value in its row and the largest value in its column are equal. It is also known as equilibrium point in game theory.

**ii) Dominance rule** also known as dominance strategy in game theory states that if one strategy of a player dominates over the other strategy in all conditions, then the later strategy can be ignored.

- If all elements in a column are greater than or equal to the corresponding elements in another column, then that column is dominated.
- If all elements in a row are less than or equal to the corresponding elements in another row, then that row is dominated.

**iii) Pure strategy game** is a game whereby both players will always play just one strategy that provides them the best payoff. It may be random or drawn from a distribution as in the case of mixed strategies.

**iv) Prisoners' dilemma** is a type of non-zero sum games (Games where the gain of one player is necessarily equal to the loss of the other player) and derives its name from a case of a bank robbery where two bank robbers were given a chance of confession. If one confesses and the other does not, then the confessor would get two years and the other one ten years. If both confess, they would get eight years each. If both refuse to confess, they would receive a lesser charge of 5 years.

b) i) Given the following payoff matrix produced by KIME&CPA Associates:

We need to test if our game has a saddle point so that we can easily find the best strategy of each player.

**Table showing minimum row and maximum column**

Employees union strategies	UR Strategies			Row minimum
	S1	S2	S3	
U1	3	2	4	<b>2</b>
U2	2	4	6	<b>2</b>
U3	1	2	4	<b>1</b>
Column maximum	<b>3</b>	<b>4</b>	<b>6</b>	

Maximin=2 Minimax=3 since minimax is different from maximin, there is no saddle point. The game is a mixed strategy. We can apply the dominance rule to solve our game. This means that Players will then play each strategy for a certain proportion (fraction)of the time.

**Dominance rule:**

- If all elements in a column are greater than or equal to the corresponding elements in another column, then that column is dominated.
- If all elements in a row are less than or equal to the corresponding elements in another row, then that row is dominated.

From this assumption, column S3 is dominated by column S1 and rowU3 is dominated by U2, The reduced payoff table remain as:

**Dominance Rule Table**

Employees union strategies	UR Strategies	
	S1	S2
U1	3	2
U2	2	4

**Union Strategies with proportion of time /percentage**

Suppose the union plays strategy U1 a fraction p (>=0) of the time, then it will play strategy U2 a fraction (1-p) of the time. The gain for the union if UR plays strategy S1 is:

$$3P+2(1-P) = 3P+2-2P$$

$$= P+2$$

The gain of the union once UR plays S2 will be:

$$2P+4(1-P) = 2P+4-4P$$

$$=4-2P$$

Let V denotes the value of the game for union,

$$\left\{ \begin{array}{l} V \leq P+2 \\ V \leq 4-2P \end{array} \right. \quad P+2 = 4-2P \quad P+2-4+2P=0 \quad 3P-2=0 \quad P=2/3$$

Or  $V=4-2P \quad V=4-2(2/3)$

**Best strategy for the union:** The strategy U1 will be played 2/3 times while strategy U2 will be played 1/3 times which is (1-p), The value of the game for the union will be:

$$V=P+2 \quad v=2/3+2 \quad V=\frac{8}{3}$$

### UR Strategies with proportion of time /percentage

Assume that UR plays S1 Q proportion of times and S2 1-q proportion of times. If union plays U1 and U2 respectively, the value of the game will be as follows for UR

$$\begin{array}{llll} V=3Q+2(1-Q) & V=3Q+2-2Q & V=Q+2 & \\ V=2Q+4(1-Q) & V=4-2Q & 4-2Q=Q+2 & -3Q=-2 \quad Q=2/3 \quad 1-Q=1/3 \end{array}$$

The UR management will play the strategy S1 a proportion of 2/3 and S2 will be played 1/3.and the value of the game will be as follows.

$$V=Q+2 \quad =2/3+2 \quad V=8/3$$

ii) Let V denotes the value of the game for union:

$$\left[ \begin{array}{l} V \leq P+2 \\ V \leq 4-2P \quad P+2 = 4-2P \quad P+2-4+2P=0 \quad 3P-2=0 \quad P=2/3 \\ \text{Or } V=4-2P \quad V=4-2(2/3) \end{array} \right.$$

The strategy U1 will be played 2/3 times while strategy U2 will be played 1/3 times which is (1-p), The value of the game for the union will be:

$$V=P+2 \quad v=2/3+2 \quad V=\frac{8}{3} \quad ****$$

Assume that UR plays S1 Q proportion of times and S2 1-q proportion of times. If union plays U1 and U2 respectively, the value of the game will be as follows for UR

$$\begin{array}{llll} V=3Q+2(1-Q) & V=3Q+2-2Q & V=Q+2 & \\ V=2Q+4(1-Q) & V=4-2Q & 4-2Q=Q+2 & -3Q=-2 \quad Q=2/3 \quad 1-Q=1/3 \end{array}$$

**Best strategy for UR:** The UR management will play the strategy S1 a proportion of 2/3 will S2 will be played 1/3.and the value of the game will be as follows.

$$V=Q+2 \quad =2/3+2 \quad V=8/3**** \quad V=\frac{8}{3} \%$$

ii) The value of the game of  $\frac{8}{3} \%$  represents a long run salary increment.

### c) Merits of purpose sampling

- Under proper safeguard, it is economical and time saving.

- In this, knowledge of composition of universe, ensure the proper representation of a cross-
- section of various strata.
- It is useful when certain units are important to be included to fulfill the requirement of
- investigation.
- It is practicable, when randomization is not possible

### Marking guide

#### QUESTION SIX

Marks

a)	i) Exponential smoothing for MUGEMA Ltd	0.5
	Stating the formula for exponential smoothing	0.5
	Calculation of the forecasts for each month (0.5 each, max 6.5)	6.5
	ii) Uses/importance of time series analysis (1 each, max 4)	4.0
	<b>Maximum marks</b>	<b>11</b>
b)	i) Calculation of marginal cost	2.0
	(ii) Computation of output that maximizes revenue	2.0
	Computation of maximum revenue	1.0
	(iii) Finding of total profit function	1.0
	Derivative of profit	1.0
	Computation of output that maximizes profit	1.0
	Computation of maximum profit	1.0
	<b>Maximum marks</b>	<b>9.0</b>
	<b>Total marks</b>	<b>20</b>

### Model Answer

- a) i) Forecasted profits

Month	Profits (FRW “million”)	Forecasts	Workings Formula $F_t = F_{t-1} + \alpha (A_{t-1} - F_{t-1})$
January, 2021	42	<b>42</b>	42
February, 2021	40	<b>42</b>	$F_2 = 42 + 0.5*(42 - 42) = 42$
March, 2021	41	<b>41</b>	$F_3 = 42 + 0.5*(40 - 42) = 43$
April, 2021	43	<b>41</b>	$F_4 = 41 + 0.5*(41 - 41) = 41$
May, 2021	38	<b>42</b>	$F_5 = 41 + 0.5*(43 - 41) = 42$
June, 2021	43	<b>40</b>	$F_6 = 42 + 0.5*(38 - 42) = 40$



July, 2021	36	<b>42</b>	$F_7 = 40 + 0.5*(43 - 40) = 42$
August, 2021	39	<b>39</b>	$F_8 = 42 + 0.5*(36 - 42) = 39$
September, 2021	37	<b>39</b>	$F_9 = 39 + 0.5*(39 - 39) = 39$
October, 2021	39	<b>38</b>	$F_{10} = 39 + 0.5*(37 - 39) = 38$
November, 2021	42	<b>39</b>	$F_{11} = 38 + 0.5*(39 - 38) = 39$
December, 2021	43	<b>41</b>	$F_{12} = 39 + 0.5*(42 - 39) = 41$
January, 2022		<b>42</b>	$F_{13} = 41 + 0.5*(43 - 41) = 42$

## ii) Uses of time series

- It helps in the understanding of the past behaviour. The past trend helps in predicting the future behaviour.
- It enables us to predict or forecast the behaviour of the phenomenon in future which is very essential for business planning.
- It helps in making comparative studies in the values of different phenomenon at different times or places.
- It helps in the evaluation of current achievements.
  - The segregation and study of various components of time series is of paramount importance to a businessman in the planning of future operations and policy decisions.
  - The main objective of analysing time series is to understand, interpret and evaluate changes in economic phenomena in the hope of more correctly anticipating the course of future events.

$$i) \quad \text{Marginal cost} = \frac{dC}{dx} = \frac{d\left(\frac{x^3}{3} - 3x^2 + 9x\right)}{dx}$$

$$\text{Marginal cost} = x^2 - 6x$$

$$\text{At 100 unit, marginal cost} = (100)^2 - (6*10) = 9,400$$

**Marginal cost of Brave Ltd is FRW 9,400,000**

- ii) To get maximum revenue, we first find the derivative of total revenue.

$$R = 21x^2 - x - 16$$

$$R' = 42x - 1$$

The necessary condition is  $R' = 0$ . Therefore  $42x - 1 = 0$ .  $x = 1/42$

$$R'' = 42 > 0$$

**The revenue will be minimum**

Minimum revenue is found by substituting the value of  $x$  into the revenue function.

$$R = 21x^2 - x - 16$$

$$R = 21(1/42)^2 - 1/42 - 16 = -16.01$$

**The maximum revenue is FRW (16,012)**

iii) First find the total profit function

**Total profit function**

Profit = total revenue – total cost

$$\text{Profit} = (21x^2 - x - 16) - \left(\frac{x^3}{3} - 3x^2 + 9x\right)$$

$$\text{Profit} = 21x^2 + 3x^2 - x - 9x - 16 - \frac{x^3}{3}$$

$$\text{Profit} = 24x^2 - 10x - \frac{x^3}{3} - 16$$

**Derivative of profit**

$$P' = 48x - 10 - x^2$$

$$P' = 0 \quad x^2 - 48x + 10 = 0$$

$$\begin{aligned} \text{Using the quadratic formula } X_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-48) \pm \sqrt{(-48)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1} = \frac{48 \pm 47.58}{2} \end{aligned}$$

$$x_1 = 47.79 \quad x_2 = 0.21$$

$$P'' = 48 - 2x$$

Substitute the values of  $x$  in the second derivative of profit for the check of the output that maximizes profit

$$P'' = 48 - 2 \cdot 47.79 = -47.58 < 0. \text{ At this level of output, profit is maximized.}$$

**Output that maximizes profit is 47.79  $\approx$  48 units**

$$P'' = 48 - 2 \cdot 0.21 = 47.58 > 0. \text{ Here profit is minimized.}$$

Therefore maximum profit =  $24(47.79)^2 - (10 \times 47.79) - \frac{47.79^3}{3} - 16 = 17,969.04 \approx 17,969$ .

**The maximum profit is FRW 17,969,000**

### Marking guide

#### QUESTION SEVEN

Marks

a) Computation of median	
Stating the formula for median	0.5
Calculation for each cumulative frequency (0.5 each, max 4)	4.0
Computation of median	0.5
<b>Maximum marks</b>	<b>5.0</b>
b) Computation of standard deviation	
Stating the formula for standard deviation	1.0
Calculation for each $d_i$ and $d_i^2$ (0.5 each, max 8)	8.0
Computation of standard deviation	1.0
<b>Maximum marks</b>	<b>10</b>
c) Bar Graph	
Each bar well drawn (1 Mark each, max 5)	<b>5.0</b>
<b>Total marks</b>	<b>20</b>

#### Answer model

a) Median

Class	0-20	20-40	40-60	60-80	80-100	100-120	120-140	140-160	Total
Frequency=f	10	15	15	20	8	8	6	8	90
Cumulative frequency	10	25	40	60	68	76	82	90	

$$M_e = L_0 + \frac{h}{f_0} \left( \frac{n}{2} - F \right)$$

$$L_0 = 60, h = 20, f_0 = 20, \frac{n}{2} = 45, F = 40$$

$$\text{Therefore, Median} = 60 + \frac{20}{20} (45 - 40) = 65$$

b) Standard deviation Assumed mean  $A = 35$

Standard deviation

Items: $X_i$	Deviation: $d_i = x_i - A$	$d_i^2$
35	0	0
37	2	4
30	-5	25
33	-2	4
36	1	1
35	0	0
39	4	16
37	2	4
Sum	2	54

$$sd = \sqrt{\frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} = \sqrt{\frac{54}{8} - \frac{4}{64}} = \sqrt{\frac{328}{64}} = 2.5860$$

c) Bar chart for the data given

